

University of California, Berkeley  
Physics H7B Spring 1999 (*Strovink*)

### SOLUTION TO PROBLEM SET 1

#### 1. RHK problem 22.9

It is an everyday observation that hot and cold objects cool down or warm up to the temperature of their surroundings. If the temperature difference  $\Delta T$  between an object and its surroundings ( $\Delta T = T_{\text{obj}} - T_{\text{sur}}$ ) is not too great, the rate of cooling or warming of the object is proportional, approximately, to this difference; that is,

$$\frac{d\Delta T}{dt} = -A(\Delta T),$$

where  $A$  is a constant. This minus sign appears because  $\Delta T$  decreases with time if  $\Delta T$  is positive and increases if  $\Delta T$  is negative. This is known as *Newton's law of cooling*.

(a) On what factors does  $A$  depend? What are its dimensions?

**Solution:** The LHS (and therefore the RHS) of the above equation have dimensions  $^{\circ}\text{C}/\text{sec}$ , so  $A$  must have dimension  $\text{sec}^{-1}$ . Suppose that the heat flowing between the object and its surroundings is conducted by a thermal barrier (*i.e.* a “skin” on the object that tends to insulate it from its surroundings). Then, from RHK Eq. 25.45,  $A$  should be proportional to the thermal conductivity of that barrier and to its area, and inversely proportional to the barrier's thickness.

(b) If at some instant  $t = 0$  the temperature difference is  $\Delta T_0$ , show that it is

$$\Delta T = \Delta T_0 \exp(-At)$$

at a time  $t$  later.

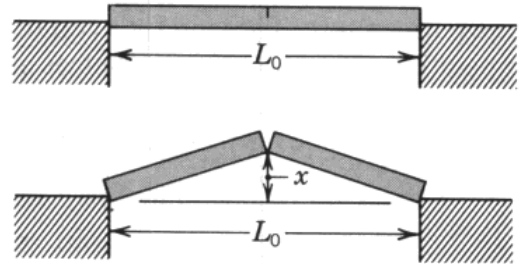
**Solution:** Rearranging and solving the above

equation, with  $dt'$  substituted for  $dt$ ,

$$\begin{aligned} \frac{d\Delta T}{\Delta T} &= -A dt' \\ \int_0^t \frac{d\Delta T}{\Delta T} &= -\int_0^t A dt' \\ \ln(\Delta T(t)) - \ln(\Delta T(0)) &= -At \\ \ln\left(\frac{\Delta T(t)}{\Delta T(0)}\right) &= -At \\ \frac{\Delta T(t)}{\Delta T(0)} &= \exp(-At) \\ \Delta T(t) &= \Delta T_0 \exp(-At). \end{aligned}$$

#### 2. RHK problem 22.28

As a result of a temperature rise of  $32^{\circ}\text{C}$ , a bar with a crack at its center buckles upward, as shown in the figure. If the fixed distance  $L_0 = 3.77$  m and the coefficient of linear thermal expansion is  $25 \times 10^{-6}$  per  $^{\circ}\text{C}$ , find  $x$ , the distance to which the center rises.



**Solution:** In Physics H7B, all problems involving numbers should be solved completely in terms of algebraic symbols before any numbers are plugged in (otherwise it is much more difficult to give part credit). Let

$L_0$  = fixed distance = 3.77 m

$x$  = distance to which the center rises

$L$  = thermally expanded total length of the buckled bar (twice the hypotenuse of the right triangle whose legs are  $x$  and  $L_0/2$ )

$\alpha$  = coefficient of linear thermal expansion =  $25 \times 10^{-6}$  per  $^{\circ}\text{C}$

$\Delta T$  = temperature rise =  $32^{\circ}\text{C}$

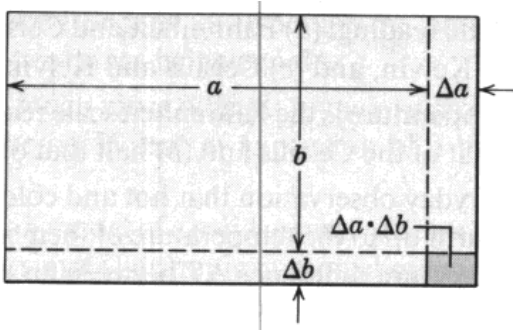
Then

$$\begin{aligned}
 L &= L_0 + \alpha \Delta T \\
 x^2 &= (L/2)^2 - (L_0/2)^2 \\
 x &= \frac{L_0}{2} \sqrt{(1 + \alpha \Delta T)^2 - 1} \\
 &= 0.0754 \text{ m} .
 \end{aligned}$$

### 3. RHK problem 22.30

The area  $A$  of a rectangular plate is  $ab$ . Its coefficient of linear thermal expansion is  $\alpha$ . After a temperature rise  $\Delta T$ , side  $a$  is longer by  $\Delta a$  and side  $b$  is longer by  $\Delta b$ . Show that if we neglect the small quantity  $\Delta a \Delta b / ab$  (see the figure), then

$$\Delta A = 2\alpha A \Delta T .$$



**Solution:** Let

$A$  = original area of rectangular plate

$a$  = original width of plate

$b$  = original height of plate

$A + \Delta A$  = thermally expanded area of plate

$a + \Delta a$  = thermally expanded width of plate

$b + \Delta b$  = thermally expanded height of plate

$\alpha$  = coefficient of linear thermal expansion

Then

$$\begin{aligned}
 A + \Delta A &= (a + \Delta a)(b + \Delta b) \\
 &= ab + a \Delta b + b \Delta a + \Delta a \Delta b
 \end{aligned}$$

$$A = ab$$

$$A + \Delta A - A = ab + a \Delta b + b \Delta a + \Delta a \Delta b - ab$$

$$\Delta A = a \Delta b + b \Delta a + \Delta a \Delta b$$

$$= ab \left( \frac{\Delta b}{b} + \frac{\Delta a}{a} + \frac{\Delta a \Delta b}{ab} \right)$$

$$\Delta A \approx ab \left( \frac{\Delta b}{b} + \frac{\Delta a}{a} \right)$$

$$\frac{\Delta a}{a} = \frac{\Delta b}{b} = \alpha \Delta T$$

$$\Delta A \approx A(\alpha \Delta T + \alpha \Delta T)$$

$$\frac{\Delta A}{A} \approx 2\alpha \Delta T .$$

### 4. RHK problem 25.47

The average rate at which heat flows out through the surface of the Earth in North America is  $54 \text{ mW/m}^2$ , and the average thermal conductivity of the near surface rocks is  $2.5 \text{ W/m}\cdot\text{K}$ . Assuming a surface temperature of  $10^\circ\text{C}$ , what should be the temperature at a depth of  $33 \text{ km}$  (near the base of the crust)? Ignore the heat generated by radioactive elements in the crust; the curvature of the Earth can also be ignored.

**Solution:** Let

$H/A$  = heat flow per unit area through Earth's surface =  $54 \times 10^{-3} \text{ W/m}^2$

$k$  = thermal conductivity of near surface rock =  $2.5 \text{ W/m}\cdot\text{K}$

$T_0$  = temperature at earth's surface =  $10^\circ\text{C}$

$D$  = depth at which we wish to know the temperature =  $33 \times 10^3 \text{ m}$

$T$  = temperature at depth  $D$

Then, using RHK Eq. 25.45,

$$\begin{aligned}
 \frac{H}{A} &= k \frac{\Delta T}{\Delta x} \\
 &= k \frac{T - T_0}{D}
 \end{aligned}$$

$$\frac{H}{A} \frac{D}{k} = T - T_0$$

$$T_0 + \frac{H}{A} \frac{D}{k} = T$$

$$723^\circ\text{C} = T .$$

**5. RHK problem 25.50**

A cylindrical silver rod of length 1.17 m and cross-sectional area  $4.76 \text{ cm}^2$  is insulated to prevent heat loss through its surface. The ends are maintained at temperature difference of  $100^\circ\text{C}$  by having one end in a water-ice mixture and the other in boiling water and steam.

(a) Find the rate (in W) at which heat is transferred along the rod.

**Solution:** Let

$L$  = length of cylindrical silver rod = 1.17 m

$A$  = area of rod =  $4.76 \times 10^{-4} \text{ m}^2$

$k$  = thermal conductivity of silver =  $428 \text{ W/m}\cdot\text{K}$

$\Delta T$  = temperature difference between ends of rod =  $100^\circ\text{C}$ .

$H = dQ/dt$  = rate at which heat is transferred along the rod.

Then, using RHK Eq. 25.45

$$\begin{aligned} H &= kA \frac{\Delta T}{L} \\ &= kA \frac{\Delta T}{L} \\ &= 17.4 \text{ W} . \end{aligned}$$

(b) Calculate the rate (in kg/sec) at which ice melts at the cold end.

**Solution:** Let

$L_f$  = latent heat of fusion of water =  $333 \times 10^3 \text{ J/kg}$

$dm/dt$  = rate in kg/sec at which ice melts at the cold end

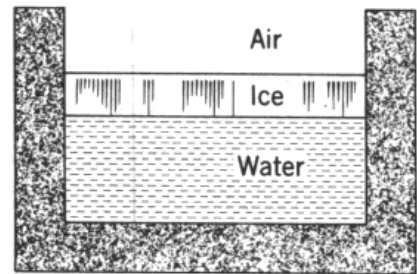
Then, using RHK Eq. 25.7,

$$\begin{aligned} Q &= L_f m \\ \frac{dQ}{dt} &= L_f \frac{dm}{dt} \\ H &= L_f \frac{dm}{dt} \\ \frac{H}{L_f} &= \frac{dm}{dt} \\ 5.23 \times 10^{-5} \text{ kg/sec} &= \frac{dm}{dt} . \end{aligned}$$

**Hints:** The thermal conductivity of silver is  $428 \text{ W/m}\cdot\text{K}$ . The latent heat of fusion of water is  $333 \text{ kJ/kg}$ .

**6. RHK problem 25.58**

A container of water has been outdoors in cold weather until a 5.0-cm-thick slab of ice has formed on its surface (see the figure). The air above the ice is at  $-10^\circ\text{C}$ . Calculate the rate of formation of ice (in centimeters per hour) on the bottom surface of the ice slab. Take the thermal conductivity and density of ice to be  $1.7 \text{ W/m}\cdot\text{K}$  and  $0.92 \text{ g/cm}^3$ , respectively. Assume that no heat flows through the walls of the tank.



**Solution:** Let

$A$  = area of slab of ice on water's surface

$h$  = present thickness of slab =  $0.05 \text{ m}$

$T$  = temperature of air above ice =  $-10^\circ\text{C}$

$T_0$  = temperature at which water freezes =  $0^\circ\text{C}$

$k$  = thermal conductivity of ice =  $1.7 \text{ W/m}\cdot\text{K}$

$\rho$  = density of ice =  $0.92 \times 10^3 \text{ kg/m}^3$

$L_f$  = latent heat of fusion of water =  $333 \times 10^3 \text{ J/kg}$

$H = dQ/dt$  = heat flow (in W) through the ice  
 $dm/dt$  = rate of formation of ice (in kg/sec) on the bottom surface of the slab

$dh/dt$  = rate of change of ice thickness (in m/sec).

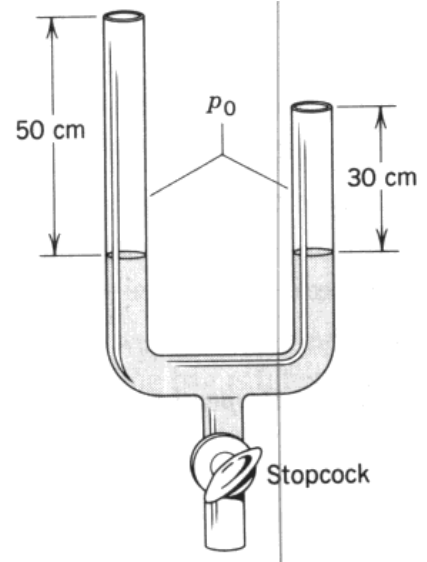
Then, using RHK Eqs. 25.45 and 25.7,

$$\begin{aligned}
 H &= kA \frac{\Delta T}{\Delta x} \\
 &= kA \frac{T_0 - T}{h} \\
 Q &= L_f m \\
 \frac{dQ}{dt} &= L_f \frac{dm}{dt} \\
 H &= L_f \frac{dm}{dt} \\
 kA \frac{T_0 - T}{h} &= L_f \frac{dm}{dt} \\
 \rho h A &= m \\
 \frac{dh}{dt} &= \frac{1}{\rho A} \frac{dm}{dt} \\
 &= \frac{1}{\rho A L_f} L_f \frac{dm}{dt} \\
 &= \frac{kA}{\rho A L_f} \frac{T_0 - T}{h} \\
 &= \frac{k}{\rho L_f} \frac{T_0 - T}{h} \\
 &= 1.11 \times 10^{-6} \text{ m/sec} \\
 &= 0.400 \text{ cm/hr} .
 \end{aligned}$$

Note that the inverse dependence of  $dh/dt$  upon  $h$  requires  $h$  to increase only as the square root of the time  $t$ . Our numerical result for the rate of growth of the ice thickness is valid only when the ice has a particular thickness (5 cm).

### 7. RHK problem 23.16

A mercury-filled manometer with two unequal-length arms of the same cross-sectional area is sealed off with the same pressure  $p_0$  of perfect gas in the two arms (see the figure). With the temperature held constant, an additional  $10.0 \text{ cm}^3$  of mercury is admitted through the stopcock at the bottom. The level on the left increases  $6.00 \text{ cm}$  and that on the right increases  $4.00 \text{ cm}$ . Find the original pressure  $p_0$ .



**Solution:** Let

$\rho$  = density of Hg =  $13.6 \times 10^3 \text{ kg/m}^3$

$g$  = acceleration of gravity at earth's surface =  $9.81 \text{ m/sec}^2$

$L_0$  = initial height of gas in left arm of manometer =  $0.50 \text{ m}$

$R_0$  = initial height of gas in right arm of manometer =  $0.30 \text{ m}$

$L$  = final height of gas in left arm of manometer =  $0.44 \text{ m}$

$R$  = final height of gas in right arm of manometer =  $0.26 \text{ m}$

$A$  = cross-sectional area of each manometer arm

$p_0$  = initial pressure in both arms of manometer

$p_L$  = final pressure in left arm of manometer

$p_R$  = final pressure in right arm of manometer

$N_L$  = no. of gas molecules in left arm of manometer

$N_R$  = no. of gas molecules in right arm of manometer

$k_B$  = Boltzmann's constant

$T$  = (constant) temperature

Applying the perfect gas law,

$$p_0 A L_0 = N_L k_B T$$

$$p_0 A R_0 = N_R k_B T$$

$$p_L A L = N_L k_B T$$

$$p_R A R = N_R k_B T$$

$$p_0 L_0 = p_L L$$

$$p_0 R_0 = p_R L$$

$$p_0 \frac{L_0}{L} = p_L$$

$$p_0 \frac{R_0}{R} = p_R$$

$$(I) \quad p_R - p_L = p_0 \left( \frac{R_0}{R} - \frac{L_0}{L} \right) .$$

Using Archimedes' principle (first equation on RHK page 387), the difference  $(L_0 - L) - (R_0 - R)$  in final height of Hg between the two arms is proportional to the final pressure difference:

$$A(p_R - p_L) = \rho g A((L_0 - L) - (R_0 - R))$$

$$(II) \quad p_R - p_L = \rho g((L_0 - L) - (R_0 - R)) .$$

Combining equations (I) and (II),

$$p_0 \left( \frac{R_0}{R} - \frac{L_0}{L} \right) = \rho g((L_0 - L) - (R_0 - R))$$

$$p_0 = \rho g \frac{(L_0 - L) - (R_0 - R)}{R_0/R - L_0/L}$$

$$= 1.526 \times 10^5 \text{ Pa}$$

$$= 1.506 \text{ atm} .$$